

Bruchgleichungen zweiten Grades

Voraussetzungen:

• Bruchrechnung: $\frac{a \pm c}{b \pm b} = \frac{a \pm c}{b}$ $\frac{a \pm c}{b \pm d} = \frac{ad \pm bc}{bd}$ $\frac{a}{b} \cdot \frac{b}{d} = \frac{ab}{bd}$ $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

- Binomische Formeln:

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (a-b)^2 = a^2 - 2ab + b^2 \quad (a+b)(a-b) = a^2 - b^2$$

• pq – Formel: $x^2 + px + q = 0 \quad \Rightarrow \quad x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$

- Satz des Vieta:

$$x^2 + px + q = 0 \quad \Leftrightarrow \quad (x+x_1)(x+x_2) = 0, \text{ wobei } x_1 \cdot x_2 = q \quad \wedge \quad x_1 + x_2 = -p$$

Aufgaben:

Bestimmen Sie jeweils Definitions- und Lösungsmenge.

1. $\frac{1}{4-x} - \frac{2}{x^2-1} = \frac{x-6}{x^2-3x-4}$

2. $\frac{3x+4}{x-5} = \frac{2x^2-13x+27}{x^2-12x+35} + \frac{x-9}{x-7}$

3. $\frac{x}{x-2} - \frac{x-2}{x-3} = \frac{2}{5x-x^2-6}$

4. $\frac{4x-5}{3x+3} - \frac{11x^2-69x+58}{15x^2-15} = -\frac{3x+4}{5-5x}$

5. $\frac{5x-17}{4x-3} + \frac{7(x-4)}{4x-5} = 3 + \frac{12(10x+73)}{32x-16x^2-15}$

6. $\frac{3x+7}{4x-20} + \frac{4x-3}{25-5x} = \frac{11-7x}{2x-10} - 10\frac{13}{20}$

7. $\frac{a+x}{b-x} - \frac{b-x}{a+x} = \frac{2a^2-2b^2}{ab-ax+bx-x^2}$

8. $\frac{5x+2}{x^2-3x-10} + \frac{x+1}{x^2-8x+15} = \frac{x-2}{x^2-x-6}$

Lösungen:

$$1. \quad D = \mathbb{R} \setminus \{-1; 1; 4\} \quad L = \left\{ -\frac{1}{2}; 3 \right\}$$

$$2. \quad D = \mathbb{R} \setminus \{5; 7\} \quad L = \{10\}$$

$$3. \quad D = \mathbb{R} \setminus \{2; 3\} \quad L = \{ \} \quad (\text{Lösung } x=2 \notin D)$$

$$4. \quad D = \mathbb{R} \setminus \{-1; 1\} \quad L = \{15\}$$

$$5. \quad D = \mathbb{R} \setminus \left\{ \frac{3}{4}; \frac{5}{4} \right\} \quad L = \{100\}$$

$$6. \quad D = \mathbb{R} \setminus \{5\} \quad L = \{4\}$$

$$7. \quad D = \mathbb{R} \setminus \{-a; b\} \quad L = \left\{ \frac{a-b}{2} \right\}$$

$$8. \quad D = \mathbb{R} \setminus \{-2; 3; 5\} \quad L = \left\{ -\frac{7}{5}; 2 \right\}$$